

Bimerons in Double Layer Quantum Hall Systems

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Abstract

In this paper we discuss bimeron pseudo spin textures for double layer quantum hall systems with filling factor $\nu = 1$. Bimerons are excitations corresponding to bound pairs of merons and anti-merons. Bimeron solutions have already been studied at great length by other groups by minimising the microscopic Hamiltonian between microscopic trial wavefunctions. Here we calculate them by numerically solving coupled nonlinear partial differential equations arising from extremisation of the effective action for pseudospin textures. We also calculate the different contributions to the energy of our bimerons, coming from pseudospin stiffness, capacitance and coulomb interactions between the merons. Apart from augmenting earlier results, this allows us to check how good an approximation it is to think of the bimeron as a pair of rigid objects (merons) with logarithmically growing energy, and with electric charge $\frac{1}{2}$. Our differential equation approach also allows us to study the dependence of the spin texture as a function of the distance between merons, and the inter layer distance. Lastly, the technical problem of solving coupled nonlinear partial differential equations, subject to the special boundary conditions of bimerons is interesting in its own right.

I. INTRODUCTION

The fascinating discoveries of the quantum Hall effect (QHE) originally found in single two dimensional electron layers, have also been extended to double layer systems , thanks to the development of techniques for growing GaAs heterostructures containing two separated layers of two-dimensional electron gas (see for example references [1]). Apart from finding plateaus in Hall conductivity at total filling fractions ν corresponding to the "direct sum" of the familiar integral and odd-denominator fractional QHE in the individual single layers, experiments also show the occurrence of new plateaus which are intrinsic to double-layer systems and rely on interlayer quantum coherence and correlations. On the theoretical front, a large body of work has already been done on double-layer systems. An extensive list of references to this literature has been given in the lucid review of this subject by Girvin and MacDonald [2] and in the paper by Moon *etal* [3].

Generally one analyses double layer systems by attributing to the electrons, in addition to their spatial coordinates on the plane, a two-component "pseudospin" whose up and down components refer to the amplitude for the electron to be in the first and second layers respectively. The real physical spin of the electrons is assumed , as a starting approximation, to be fully polarised by the strong magnetic field and hence frozen as a degree of freedom. However, even when real physical spin is suppressed, the use of a pseudospin to represent the layer degree of freedom maps the double layer spinless problem into a monolayer problem with spin [4]. Such a mapping allows one to borrow for double layer systems, the rich body of insights and results available from single layer systems with real spin.

Thus one may expect a fully symmetric (polarised) pseudospin state to be energetically preferred because of a combination of Coulomb repulsion and the Pauli principle which forces an associated antisymmetric spatial wavefunction, just as in itinerant ferromagnetism. Further, the relevance of Skyrmions to systems with real spin, predicted by theoretical considerations [5], [6] and supported by experimental evidence [7], has in turn prompted studies of similar topological excitations in spinless double layer systems, but now involving

pseudospin (See Girvin and MacDonald [2], Moon *etal* [3] and references given therein).

Because of interplane-intraplane anisotropy in Coulomb repulsion between electrons located in the two layers, as well the capacitance energy of maintaining unequal charge density in the two layers, the effective Action governing pseudospin enjoys only U(1) symmetry of rotations about the z-axis (the direction perpendicular to the x-y plane of the layers). Finiteness of the capacitance energy between the two layers requires that asymptotically the pseudospin must lie on the easy (x-y) plane. The basic topological excitations in that case are the so-called merons which are vortices in pseudospin with a winding number of one-half (with respect to the second Homotopy group Π_2 . These are similar to vortices in the X-Y model, but non singular at the origin since the pseudospin is not restricted to lie on the x-y plane. But like the former they do have an energy that grows logarithmically with size. One can also have meron anti-meron bound pairs whose energy is finite. Such a pair is topologically equivalent to Skyrmions and carries unit winding number. (For an introduction to such topological excitations, their winding numbers, etc. see reference [8].)

The possibility of topological excitations like merons and bimerons in double layer systems has generated much interest, in part because of the excitement surrounding the Skyrmion excitations in systems with real spin, and in part because of the additional possibility here of Kosterlitz-Thoules type [9] phase transitions caused by the break-up of bound bimerons into separated meron pairs [2], [3]. Bimeron solutions have already been extensively studied in a body of papers by Girvin, MacDonald and co-workers [10] [11] and [3]. These calculations are based on optimising microscopic wavefunctions with respect to the microscopic interaction Hamiltonian.

We will also calculate bimeron solutions and their energies here, but by using an alternate method. An Effective Action for slowly varying pseudospin textures has already been obtained by Moon et al [3]. If one extremises that Action one will get differential equations which the unit-vector valued field of pseudo spin configurations $\vec{m}(\vec{r})$ should obey in the classical limit.

In this paper we solve these coupled non-linear differential equations, through a com-

bination of analytically motivated ansatze followed by numerical calculations. We obtain bimerons as approximate time-independent solutions with appropriate topologically non-trivial boundary conditions, for a range of separations between the meron and its partner the anti-meron and also for a set of different inter-layer distances. The dependence of the bimeron texture on these variables is discussed. They turn out to be reasonably similar to what one would expect on general grounds. We also obtain the energy of this bimeron as a function of the separation between the meron centers. We include in this energy contributions coming from the pseudospin stiffness, its anisotropy, the capacitance energy and the Coulomb energy. By minimising this energy with respect to the meron separation, we are also able to give an independent value for the optimal meron separation in a bimeron. We compare these results with earlier work, including our own.

Apart from this, our work also enables us to independently check the validity of a physical picture often used [11] in estimating bimeron energies, namely, that they can be viewed as a pair of rigid objects carrying electric charge of $\frac{1}{2}$ and a logarithmically growing energy. A work somewhat similar in spirit to ours, but in the context of Skyrmons of real spin systems was done by Abolfath *et.al.* who compared results obtained from solving a non-linear differential equation with those obtained from microscopic calculations [12]. For yet another way of approaching meron solutions, starting from a Chern- Simons field theory see the work of Ichinose and Sekiguchi [13].

In an earlier paper [14] we had done a similar study of single meron solutions. But the present work is much more complicated at the computational level. Single meron solutions are circularly symmetric, with the spin component on the plane pointing along the coordinate direction. Thus the only unknown, namely, the spin-z component obeys an ordinary (though non-linear) differential equation in the radius variable r . Further, the boundary conditions relevant to a single meron can be imposed, conveniently, at the end points $r = 0$ and $r = \infty$. By contrast the boundary conditions characterising a bimeron are $m_z = \pm 1$ at two finite points on the plane where the two merons have their centers. The spin direction is also not simply related to the coordinate direction, so that there are two independent fields, say, m_z

and $\tan^{-1}\left(\frac{m_x}{m_y}\right)$, (since the \vec{m} is constrained to be a unit vector) which obey coupled partial differential equations on the plane. We found it quite challenging to analyse these coupled equations analytically as far as possible, and use that information to employ an appropriate ansatz and coordinate system to numerically solve the equations (using a desk-top computer).

Finally, we should reiterate that our work here clearly relies heavily on the advances already made by the Indiana group [3], [11], [10] and is to be viewed as something which will hopefully augment their findings.

II. THE SPIN TEXTURE EQUATIONS .

The differential equations obeyed by spin textures is obtained by extremising an effective action which has already been derived by Moon *et al* [3] starting from the basic microscopic physics. See also Ezawa [16]. These results were summarised in our earlier paper [14]. Briefly , the pseudospin texture of a state is described by a classical *unit* vector $\vec{m}(\vec{r})$ which gives the local direction of the pseudospin. Here \vec{r} is the coordinate on the x-y plane carrying the layers, while the magnetic field B is along the z-direction. The fully polarised "ferromagnetic" ground state corresponds to \vec{m} pointing everywhere in the same direction, say, along the x-axis. Using this as the reference state, any other state with some arbitrary texture $\vec{m}(\vec{r})$ is given by performing a local pseudospin rotation on this uniform ground state . The leading low-wavelength terms in the effective Action for time independent configurations $\vec{m}(\vec{r})$, as obtained by Moon *et al* [3] is

$$I(\vec{m}) = \int d^2r \left[\frac{1}{2} \rho_A (\nabla m_z)^2 + \frac{1}{2} \rho_E ((\nabla m_x)^2 + (\nabla m_y)^2) + \beta m_z^2 \right] + C_1[m] + C_2[m] \quad (2.1)$$

where

$$C_1[\mathbf{m}] = \frac{1}{2} \int d\vec{r} d\vec{r}' V(\vec{r} - \vec{r}') q(\vec{r}) q(\vec{r}') \quad (2.2)$$

and

$$C_2[\mathbf{m}] \equiv \frac{e^2 d^2}{32 \pi^2 \epsilon} \int d^2r \int d^2r' \left(\frac{m_z(\mathbf{r}) \nabla^2 m_z(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \right) \quad (2.3)$$

The constants ρ_A and ρ_E are pseudospin stiffness parameters whose physical origin is the exculsion principle (Hund's rule) mentioned earlier. They are given by

$$\begin{aligned}\rho_A &= \left(\frac{\nu}{32\pi^2}\right) \int_0^\infty dk k^3 V_k^A \exp\left(\frac{-k^2}{2}\right) \\ \rho_E &= \left(\frac{\nu}{32\pi^2}\right) \int_0^\infty dk k^3 V_k^E \exp\left(\frac{-k^2}{2}\right)\end{aligned}\quad (2.4)$$

where $V_k^A = 2\pi e^2/(\epsilon k)$ and $V_k^E = (\exp(-kd)2\pi e^2)/(\epsilon k)$ are the Fourier transforms of the Coulomb interactions between electrons in the same and different layers respectively. All distances (and inverse wave vectors) are in units of the magnetic length l . The βm_z^2 term represents the so-called capacitance or charging energy needed to maintain unequal amounts of charge density in the two layers. Recall that the z-component of pseudospin represents the difference between the densities in the two layers. The constant β is given by

$$\beta = \left(\frac{\nu}{8\pi^2}\right) \int_0^\infty dk k (V^z(0) - V^z(k)) \exp\left(\frac{-k^2}{2}\right) \quad (2.5)$$

where $V_k^z = \frac{1}{2}(V_k^A - V_k^E)$. Finally, $q(\vec{r})$ is the topological density associated with pseudospin texture, which is also its charge density [5]. It is given by

$$q(\vec{r}) = -\frac{\nu}{8\pi} \epsilon_{\nu\mu} \mathbf{m}(\vec{r}) \cdot [\partial_\nu \mathbf{m}(\vec{r}) \times \partial_\mu \mathbf{m}(\vec{r})] \quad (2.6)$$

We will refer to the the non-local term C_1 , as the Coulomb term since it has been identified as the Coulomb energy associated with topological structures in the pseudospin textures [3], [5]. The other non local term C_2 arises in the gradient expansion but is not amenable to simple physical interpretation.

The field equations are obtained by extremising this Hamiltonian with respect to the independent field variables, which can be taken to be m_z and $\alpha \equiv \tan^{-1}\left(\frac{m_x}{m_y}\right)$. This α is just the azimuthal angle of the projection of \vec{m} on to the x-y plane. The non-local terms C_1 and C_2 in the Action (2.1) will render the field equations into coupled integro-differential equations. While in the single meron case we did solve such an integro differential equation [14], for the more complicated case of bimerons we will be content to solve the equations in the absence of the integral terms C_1 and C_2 . The contributions of these terms can however

be included in the total energy, but by using solutions of the local equations. In mild justification of this strategy, we will find later that the Coulomb energy C_2 for instance is less than half the energy from the local terms in eq. (2.1).

The coupled field equations for m_z and $\alpha \equiv \tan^{-1}\left(\frac{m_x}{m_y}\right)$ resulting from eq. (2.1) in the absence of C_1 and C_2 are

$$\rho_A \nabla^2 m_z + \rho_E m_z \left(\frac{(\nabla m_z)^2}{(1 - m_z^2)^2} + \frac{m_z \nabla^2 m_z}{1 - m_z^2} + \nabla^2 \alpha \right) - 2\beta m_z = 0 \quad (2.7)$$

and

$$\vec{\nabla} \cdot [(1 - m_z^2) \vec{\nabla} \alpha] = 0 \quad (2.8)$$

III. BIPOLAR COORDINATES

To find bimeron solutions we have to numerically solve the coupled partial differential equations (PDE) in (2.7) and (2.8). The defining boundary condition of a bimeron is $m_z = \pm 1$ at the points $(0, \pm a)$. Our strategy will be to use the known exact solution of these equations in the Non-Linear Sigma Model (NLSM) limit, and solve the full equations iteratively starting with the NLSM solution. The NLSM limit is realised when the layer separation d goes to zero in which case we see from their defining equations above that $\rho_A = \rho_E$, i.e. the stiffness is isotropic and further that the capacitance coefficient β vanishes. Then, with C_1 and C_2 also neglected, the action in (2.1) is just that of the NLSM, all of whose solutions are exactly known [8]. They are conveniently described by the complex field $w(z)$ which represents the stereographic projection of the unit sphere of textures \vec{m} . It is defined by

$$w(z) \equiv \frac{m_x + im_y}{(1 - m_z)} \quad (3.1)$$

where $z = x + iy$. Our texture variables m_z and α are related to $w(z)$ by

$$m_z = \frac{|w|^2 - 1}{|w|^2 + 1} \quad \text{and} \quad \alpha = \arg(w) \quad (3.2)$$

Any analytic function $w(z)$ will be a solution of the NLSM. In particular the function

$$w(z) = \frac{z - a}{z + a} \quad (3.3)$$

represents the bimeron, with the points $(0, -a)$ and $(0, a)$ representing the centers of the two merons, where the solution gives $m_z = \pm 1$ respectively. It may be checked that (3.3) satisfies the coupled equations (2.7) and (2.8) in the isotropic limit.

When the interlayer separation d is not zero, we have to cope with the coupled field equations (2.7) and (2.8) with both the anisotropic stiffness and capacitance terms present. Some analysis of this system was done long ago by Ogilvie and Guralnik [17] who studied the NLSM with the mass (capacitance) term included but no anisotropy. (An ansatz suggested in ref ([17]) does not work, as we will show below.) Meanwhile Watanabe and Otsu [18] studied the anisotropic NLSM but without the mass term. Both made considerable progress analytically, but neither offered exact or numerical solutions. Here we will try to solve (2.7) and (2.8) numerically after including both the capacitance and anisotropic terms.

To do so, it will be convenient to use a bipolar coordinate system to describe the x - y plane, as might be expected when we have to impose boundary conditions at two finite points $(0, -a)$ and $(0, a)$. These coordinates, η and ϕ , are defined by

$$\begin{aligned} \eta &\equiv \ln \left| \frac{z - a}{z + a} \right| \\ \text{and } \phi &\equiv \arg \left(\frac{z - a}{z + a} \right) \end{aligned} \quad (3.4)$$

This coordinate set has many advantages [19]. The points $(0, -a)$ and $(0, a)$ at which we have to impose boundary conditions are now mapped into $\eta \rightarrow \pm\infty$. The full x - y plane is mapped in (η, ϕ) coordinates to an infinite strip with $\eta = [-\infty, +\infty]$ and $\phi = [-\pi, \pi]$. Finally, it is clear upon comparing eq(3.4) to eq (3.3) that this set of coordinates is closely related to the exact NLSM bimeron solution. Clearly the the exact NLSM solution (3.3) corresponds to the simple expressions

$$\begin{aligned} m_z &= \tanh \eta \\ \text{and } \alpha &= \phi \end{aligned} \quad (3.5)$$

Away from the NLSM limit, since this is an orthogonal coordinate system with simple expressions for the gradient, divergence and Laplacian, the equations (2.7) and (2.8) become

$$\left[\left(\frac{\rho_A - \rho_E}{\rho_E} \right) + \frac{1}{1 - m_z^2} \right] (\partial_\eta^2 m_z + \partial_\phi^2 m_z) + \frac{m_z (\partial_\eta m_z + \partial_\phi m_z)^2}{(1 - m_z^2)^2} + m_z ((\partial_\eta \alpha + \partial_\phi \alpha)^2 - \frac{2\beta}{\rho_E} Q^2 (\eta, \phi)) = 0 \quad (3.6)$$

$$(1 - m_z^2)(\partial_\eta^2 \alpha + \partial_\phi^2 \alpha) - 2m_z(\partial_\eta m_z \partial_\eta \alpha + \partial_\phi m_z \partial_\phi \alpha) = 0 \quad (3.7)$$

where

$$Q^2 (\eta, \phi) = \frac{a^2}{(\cosh \eta - \cos \phi)^2} \quad (3.8)$$

is the Jacobian of this coordinate transformation. Now let us analyse these equations as different terms are included in stages.

(a) In the NLSM limit, our exact solution has $\alpha = \phi$. Then (3.7) forces m_z to be a function of η alone, $m_z = m_z(\eta)$. Upon inserting this into the other equation (3.6) it becomes an *ordinary* non-linear differential equation. This is the advantage of this choice of coordinates. The solution can be verified to be $m_z = \tanh(\eta)$.

(b) Next let us include anisotropy ($\rho_A \neq \rho_E$), while still keeping the capacitance term zero ($\beta = 0$). Once again we can set $\alpha = \phi$, and consequently $m_z = m_z(\eta)$, which will obey again an ordinary differential equation given by

$$\left[\left(\frac{\rho_A - \rho_E}{\rho_E} \right) + \frac{1}{1 - m_z^2} \right] (\partial_\eta^2 m_z) + \frac{m_z (\partial_\eta m_z)^2}{(1 - m_z^2)^2} + m_z = 0 \quad (3.9)$$

This has no analytic solution, but can be solved relatively easily numerically, being just an ordinary differential equation in the variable η . As boundary conditions we impose $m_z = 0$ at $\eta = 0$ and $m_z = 1$ at $\eta = \infty$, (Note that the equation above is symmetric under $\eta \rightarrow -\eta$, so that we can choose the solution to be antisymmetric, i.e. $m_z(-\eta) = -m_z(\eta)$). The resulting numerical solutions for different values of layer separation d (on which the anisotropy depends), are shown in fig 1. One can see that with increasing the

layer separation, and hence increasing anisotropy in the stiffness, the pseudospin component m_z reaches its asymptotic value more slowly.

(c) Finally let us also include the capacitance term and consider the equations (3.6 and 3.7) in full. Now the ansatz $\alpha = \phi$ is no longer sustainable, in contrast to what has been suggested in ref ([17]). The substitution of the ansatz $\alpha = \phi$ in equation (3.7) would again force $\partial_\phi m_z = 0$, i.e. $m_z = m_z(\eta)$. But now this is in contradiction with equation (3.6) which has an explicit ϕ dependence through the last (capacitance) term $\frac{2\beta}{\rho_E} Q^2(\eta, \phi)$. Therefore, once one includes the capacitance term in equation (3.6) both α and m_z become functions of both η and ϕ . One has unavoidably to solve the coupled non-linear PDE for $m_z = m_z(\eta, \phi)$ and $\alpha = \alpha(\eta, \phi)$.

We do this by employing what we believe is a good ansatz for α which approximately satisfies 3.7). We then solve the other equation (3.6) numerically after inserting that ansatz for α . Our ansatz is been motivated by the following arguments. One can see from equation (3.6) that the troublesome ϕ dependent term Q^2 is negligibly small in the large η region ($Q \sim \text{sech}(\eta)$) and is most dominant in the small η region. Hence α will still approach ϕ as $\eta \rightarrow \infty$ but needs to be modified substantially in the small η region where however $m_z \ll 1$. When $m_z \ll 1$ equation (3.7) can be approximated by

$$\nabla^2 \alpha = 0 \quad (3.10)$$

This is just Laplace's equation in two dimension whose solutions are all harmonic functions.

With this in mind we choose our ansatz for α as follows :

$$\alpha = \phi - B \kappa \exp(-|\eta|) \sin(\phi) \quad (3.11)$$

where

$$\kappa \equiv \left(\frac{2\beta}{\rho_E}\right)^{\frac{1}{2}} a \quad (3.12)$$

This solves Laplace's equation and satisfies all the required boundary conditions and asymptotic behaviour, namely

$$\begin{aligned}
\alpha &\rightarrow \phi \quad \text{as} \quad \eta \rightarrow \pm\infty \\
\alpha &= 0 \quad \text{when} \quad \phi = 0 \\
\alpha &= \pi \quad \text{when} \quad \phi = \pi \\
\alpha &= \phi \quad \text{when} \quad \kappa = 0
\end{aligned} \tag{3.13}$$

. Note that the ansatz has a cusp at $\eta = 0$. This need not cause concern. Some such cusps can be expected on physical grounds and are familiar in soliton physics. The point is that each meron feels some force due to the other (Coulomb plus a logarithmic force) at arbitrary separation. We would expect them to move because of this force, and cannot strictly speaking expect a static bimeron solution to exist at arbitrary separation. But a cusp, like the one in the above ansatz, amounts to a delta function in the second derivative and can be interpreted as a external force just at $\eta = 0$ which can "hold the two merons together" at arbitrary separation. For more discussion of this point see Rajaraman and Perring and Skyrme [15] where this technique was used to get intersoliton forces between one dimensional solitons.

The constant B is chosen by minimising the energy. Substituting this ansatz in equation (3.6) we then solved it numerically subject to the boundary condition

$$\begin{aligned}
m_z &= 0 \quad \text{at} \quad \eta = 0 \\
m_z &= \pm 1 \quad \text{when} \quad \eta = \pm\infty
\end{aligned} \tag{3.14}$$

. It is sufficient to solve the equation in the first quadrant .i.e. $(\eta[0, \infty] \text{ and } \phi[0, \pi])$. For the rest of the quadrants solutions can be obtained by writing

$$\begin{aligned}
m_z(-\eta, \phi) &= -m_z(\eta, \phi) = -m_z(\eta, -\phi) \\
\alpha(-\eta, \phi) &= \alpha(\eta, \phi) = -\alpha(\eta, -\phi)
\end{aligned} \tag{3.15}$$

which is consistent with the invariance of equations (3.6) and (3.7) under the transformation $\eta \rightarrow -\eta$ and $\phi \rightarrow -\phi$.

IV. NUMERICAL PROCEDURE

Before proceeding to solve this PDE (3.6) we must take note of the fact that the last term of the equation (3.6) is singular at the point $(\eta = 0, \phi = 0)$. This point corresponds to spatial infinity on the parent x-y plane. As one moves near this point the leading singularity in the equation, coming from the Q^2 term, goes like $\frac{4\kappa^2}{(\eta^2 + \phi^2)^2}$ with other subleading singularities of the form $\frac{1}{\sqrt{(\eta^2 + \phi^2)}}$. It can be seen that this leading singularity can be offset by requiring that m_z behave as $\left[\exp - \left(\frac{2\kappa}{\sqrt{\eta^2 + \phi^2}} \right) \right] g(\eta, \phi)$, where this $g(\eta, \phi)$, is a more smooth function for which one solves numerically. This corresponds, in more familiar polar coordinates (r, θ) to writing m_z in the form $\left[\exp - \left(\frac{\kappa r}{a} \right) \right] \tilde{g}(r, \theta)$. That m_z will suffer such an exponential fall-off as $r \rightarrow \infty$ can also be inferred directly from the "mass term" $2\beta m_z$ in the original field equation (2.7). Similarly one can also verify that the cancellation of the subleading singular terms can be achieved by requiring that g has to behave like $\sqrt{(\eta^2 + \phi^2)}$ as $\eta, \phi \rightarrow 0$.

Given this functional form of m_z near the origin of the η, ϕ plane, the boundary conditions (3.14), and the ansatz (3.11) for α we solved equation (3.6) through an iterative procedure. We start with the solution for $\kappa=0$ but with full anisotropy, which can be obtained relatively easily from the ordinary differential equation (3.9). We then use this solution as input to obtain the solution for κ equal to a small number ϵ through the Newton-Raphson method [20]. The solution for ϵ is then used as input to obtain the solution for 2ϵ and so on. This procedure is repeated until one reaches the desired value of κ . The advantage of this procedure is that one can make ϵ arbitrarily small to make the Newton-Raphson method converge. In this way we obtained solutions for different values of the ansatz parameter B for each value of κ .

V. RESULTS AND DISCUSSION

Our solutions of equations (3.6) and along with the ansatz (3.11) give us the value of the pseudospin vector \vec{m} as a function of η and ϕ , or equivalently, the value of the vector-field \vec{m}

on a lattice of points on the parent x-y plane. We repeated this calculation for a set of values of the parameter B in the ansatz (3.11). We found that as one varies B starting from 0, the energy does not vary much as B goes 0 to 0.1, but then it increases sharply after $B=0.1$. This behaviour is seen to be common to all κ and all a . Hence we take B to equal 0.1. and solve the PDE for a variety of values of layer separation d , and bimeron separation a (a is actually half of the meron-antimeron separation) . Together all these solutions represent a large body of calculated data. But it is neither feasible nor very interesting to try to display it all in this paper. Instead we will try to bring out salient features of our solutions through examples.

Recall from (3.9) that in the absence of the capacitance term m_z had no ϕ - dependence. To give some feel for how the m_z varies with ϕ in the presence of the capacitance term, we plot in fig. 2 the solution $m_z(\eta)$ of equation (3.6) for a set of values for ϕ . This solution corresponds to $d = 0.7$ and $a = 3.158$. The sequence of curves shown correspond to ϕ equal to 0, 0.2π , 0.47π , and 0.94π respectively with the outermost one belonging to ϕ equal to 0. As we have discussed earlier, as η and ϕ tend to zero, the solution should damp exponentially as $\exp(-\frac{\kappa}{\sqrt{\eta^2+\phi^2}})$. Correspondingly we see in fig.2 that the low ϕ curves rise very slowly as η increases away from zero. We also give for comparison, in the form of the dotted curve, the function $\tanh(\eta)$ which is the solution in the NLSM limit. The comparison shows that the restructuring of the pseudospin texture due to the capacitance and term and anisotropy is considerable.

As an alternate representation of our results, we show in fig. 3 the projection of \vec{m} on the x-y plane, for the example of d equal to 0.7 and κ equal to 4.4. (All lengths throughout this article are in units of the magnetic length l). The length of each arrow gives the magnitude of its easy-plane projection $\sqrt{m_x^2 + m_y^2}$ and its direction gives the azimuthal angle of the projected vector, namely, $\alpha = \tan^{-1}\left(\frac{m_y}{m_x}\right)$. The plot clearly shows that our "bimeron" solution is indeed a meron-antimeron pair. Note that, as desired, \vec{m} lies along the x-axis asymptotically. This picture closely resembles the general structure obtained by Brey *et.al.* [10]. The data corresponding to all other values of d and a we studied have a similar behavior.

In fig.4 we plot the topological charge density given in eq.(2.6) as a function of η and ϕ in the presence of all the local terms in the field equations, including anisotropic ones. In viewing this figure it may be helpful to remember that large $|\eta|$ corresponds to the meron centers while $\eta = 0, \phi = 0$ corresponds to spatial infinity. $\phi = \pi$ corresponds to the line joining the two merons. As topological charge density is symmetric when either of the coordinate variable changes sign we show the contours only in the first quadrant where both η and ϕ are positive.

Next let us turn to the energetics of these bimeron solutions. In fig.5. we show how the "local" energy i.e. the contribution from the local terms in the energy functional (all terms in eq(2.1) except for C_1 and C_2) varies when one changes the separation $2a$ between the meron and antimeron centres. The appearance of a minimum is quite conspicuous and generic to all the layer separations for which the energy is calculated. The example in fig. 5 corresponds to a layer separation of 0.7.

In fig.6 we plot the Coulomb energy C_1 evaluated using our solution of the equation (3.6), as a function of the bimeron separation. The continuous curve is the best fit to our calculated points. Sometimes in the literature, a phenomenological estimate of bimeron energetics is made assuming that it can be viewed as a bound pair of two merons, each symmetrical, undistorted by the other and carrying a charge of $\frac{e}{2}$. Such a pair would have a Coulomb energy of $\frac{1}{8a}$ (in units of $\frac{e^2}{\epsilon l}$ that we are using). To see how good an approximation this simple picture is, we give in the same fig.6, in the form of a broken line the plot of this function $\frac{1}{8a}$. We see that the value of the Coulomb energy we get from the actual bimeron solution is much larger than what the simple two-charge picture would give. This is presumably because each meron is considerably squashed (polarised) by the close proximity of the other. In our earlier work on single merons [14], we had found that at the layer separation ($d = 0.7$) used in fig.6, the core-radius of individual merons is about 2, which is of the same order as the meron-separation in fig 6. In fact we can see that the gap between the two curves in fig.6 is higher for smaller a where the individual merons are squeezed together more. Of course our results, while indicative, may not be quantitatively unambiguous. For instance,

recall that our solution was obtained using only the local terms in the differential equation and the Coulomb energy was calculated by substituting this solution into the integral C_1 . The non-local Coulomb term's influence on the solution has not been included.

In fig.7 we plot the variation of three terms in the energy functional namely the contribution from the local terms (capacitance+gradient energy) C_1 and C_2 as a function of the bimeron separation. The data presented here corresponds to layer separation d equal to 0.7 but this behaviour is representative of almost all the layer separations (0.5, 0.6, 0.7 and 0.8 for which we have found solutions). The trend of all three contributions is the same for the other layer separations also with only slight changes in the slope of the curves.

Our calculations were done for different bimeron separations a , for each layer separation d . In reality, the exact solution should exist only for some optimal bimeron separation a for each value of d . One can ask if our calculations would reveal this by minimising the total energy at some particular a . To see this, we have shown in Fig. 8 the total energy at $d=0.7$ (i.e. the sum of all three contributions plotted in the fig.7) as a function of bimeron separation a . As we can see from fig.7, the total energy keeps decreasing with a , all the way to about $a = 3.2$, which is the highest value upto which we could calculate, given limitations of our computing facilities. However, the decrease is clearly levelling off and is indicative that a minimum may exist at around $a=4$ or 5. What we have done, in drawing fig.8, is to obtain a best-fit-curve of the data points upto $a=3.2$ and extrapolate that curve upto $a=4.5$. For what it is worth such extrapolation indicates a minimum at about $a=4$. This corresponds to a meron-antimeron separation of about 8, larger than what Yang and MacDonald found by entirely different methods (see their fig. 2) [11]. Their value of the meron separation for $d = 0.7$ is about 4.5. We attribute this discrepancy to the fact, noted already in our discussion of fig.6, that the Coulomb energy in our explicit calculation of the bimeron solution is higher than the undistorted meron pair estimate used in ref([11]). The actual larger Coulomb repulsion is, we believe responsible for the larger optimal meron separation that we get.

We saw that the Coulomb interaction energy between the two merons as given by the

term C_1 in the present calculation differs quite a bit from the simple picture of the bimeron as a pair of undistorted merons of charge $\frac{e}{2}$ each. One can ask if there is a similar discrepancy in the non-Coulombic energy as well. This is the subject of Table 1. In the picture of a bimeron as a pair of merons [3] , [11], [14], it will have energy equal to

$$E_{prev} \equiv 2E_{mc} + 2\pi\rho_E \ln\left(\frac{2a}{R_{mc}}\right) \quad (5.1)$$

where E_{mc} and R_{mc} are respectively the core energy and radius of a single merons, which have a logarithmic interaction with each other because of the logarithmic divergence of the self energy of single merons. (As stated already we are leaving out their Coulomb interaction in the comparison being done in this table.) This E_{prev} has been calculated in our previous work [14]. It can be compared with the local part of the energy in the present calculation. Such a comparison is given in Table 1 for different values of d , using the optimal value of the meron separation a which minimises E_{local} . We see that the comparison is not bad considering the completely different ways of estimating this energy in this paper and in earlier literature.

In conclusion, our solution for the bimeron obtained by directly solving the coupled partial differential equations that the bimeron texture obeys provides an alternate way of obtaining the profiles and energies of these objects. As far as the local part of the energy is concerned, the results are in broad agreement with microscopic derivations earlier. But the Coulomb energy we obtain is higher by a factor of about 2 from earlier simple estimates because in actuality, the two merons in close proximity will not behave like undistorted symmetrical merons.

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FIGURES

FIG. 1. The solution $m_z(\eta)$ of equation (3.9). The three continuous curves correspond, as you go outwards, to three different values of layer separation d equal to 0.5, 0.6 and 0.7 respectively in the unit of magnetic length l . The dotted curve corresponds to the exact solution of NLSM i.e. $m_z = \tanh(\eta)$.

FIG. 2. The solution $m_z(\eta)$ of equation (3.6) for a set of values for ϕ . The curves correspond, as you go inwards, to $\phi = 0, 0.2\pi, 0.47\pi, 0.94\pi$ respectively with the outermost one corresponds to ϕ equal to 0. The layer separation d is equal to $0.7l$ and bimeron separation a is equal to $3.158l$. The dotted curve at the top again corresponds to $m_z = \tanh(\eta)$.

FIG. 3. This figure gives the magnitude and direction of x-y projection of \mathbf{m} at different points on the plane. The layer separation and the bimeron separation are same as in fig. 2.

FIG. 4. A contour plot of the topological charge density of the bimeron when both the capacitance term and the anisotropy term is incorporated. This particular plot corresponds to a layer separation d equal to 0.7 and bimeron separation a equal to 3.158 both in the unit of magnetic length l . The number against each contour (shown by broken curves) denotes the corresponding charge density.

FIG. 5. This figure gives the plot of the energy (E_{local}) coming from the local terms in the action as a function of bimeron separation a in the unit of magnetic length l . The unit of energy is $\frac{e^2}{\epsilon l}$. The points correspond to the actually computed values of the energy while the continuous curve is the best fitted curve to it. The form of the best-fit curve is $E = A + B(a - C)^2$ where A and B and C are found out to be .223, .008 and 2.76 respectively. This data corresponds to a layer separation d equal to $0.7l$.

FIG. 6. This figure gives the plot of the coulomb energy as a function of bimeron separation a in units of magnetic length l . The unit of energy is $\frac{e^2}{\epsilon l}$. The upper curve is our computed value of the coulomb energy integral C_1 using the solution of equation (3.6)(points). The continuous line is the best curve to these points. The form of the best fitted curve is $E = \frac{A}{a^B}$ where A and B are found out to be 0.847 and 0.821 respectively. The dotted curve at the bottom corresponds to the Coulomb energy that the bimeron would have, if viewed as a bound pair of two point charges of $\frac{e}{2}$ each, separated by a distance $2a$. This data corresponds to a layer separation d equal to 0.7

FIG. 7. this figure gives a relative estimate of the contribution of the three type of terms in the action, namely, the local terms, C_1 and C_2 , as a function of bimeron separation a . The units are as specified in the earlier figures. This data also corresponds to a layer separation $0.7l$

FIG. 8. A plot of the total energy $E(total)$ as a function of bimeron separation a , for a layer separation of 0.7. This curve was obtained by extrapolating the curve fitted to the calculated values going upto $a = 3.2$.

Table 1: The optimal bimeron separation (a), the bimeron local energy(E_{local}) and meron pair energy (E_{prev}) from our previous work [14] as a function of the layer separation d . The unit of energy is $\frac{e^2}{\epsilon l}$ and the unit of length is l

d	a	E_{local}	E_{prev}
0.5	3.30	.270	.217
0.6	3.16	.248	.226
0.7	2.72	.223	.224
0.8	2.39	.201	.214

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